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SHEARING-STRESS MEASUREMENTS BY USE OF A HEATED ELEMENT

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SUMMARY

The rate of local heat transfer from a solid surface to a moving fluid is related to the local skin friction. Measurements of heat transfer from small elements embedded in the surface of a solid can thus be used to obtain local skin-friction coefficients. This method was applied by Fage and Falkner for laminar boundary layers and by Ludwieg for turbulent boundary layers. The present report discusses the possible range of application of such an instrument in low- and high-speed flow and presents experimental data to show that a very simple instrument can be used to obtain laminar and turbulent skin-friction coefficients with a single calibration. The instrument consists of an ordinary hot-wire cemented into a groove in the surface. The heat loss from the wire is proportional to the cube root of the wall shearing stress, and the constant of proportionality may be found by one calibration, for example, in laminar flow.

INTRODUCTION

It is the purpose of the present report to discuss some aspects of the method of obtaining skin-friction coefficients from a measurement of the local rate of heat transfer. This study developed logically from the previous work on skin friction at the Guggenheim Aeronautical Laboratory of the California Institute of Technology. A compact and sensitive skin-friction balance was developed by Dhawan (ref. 1) and further improved by Coles (ref. 2) and Hakkinen (unpublished). Coles' extensive measurements of local skin friction in supersonic flow show that very accurate measurements can be obtained using such a balance. The force measurements have two great advantages. First, they are absolute measurements; that is, no calibration other than the determination of the spring constant of the system is necessary. Second, the result of the measurement yields the shearing stress directly. The method has the disadvantage that a rather delicate mechanism is employed and furthermore is essentially restricted to cases of zero, or at least small, pressure gradients.

Coles (ref. 2) has employed the balance in flows with a moderate pressure gradient and has shown that the instrument can be calibrated for this purpose. However, the measurement requires a correction for apparent shear due to pressure force, and hence one of the main advantages of the floating-element technique is lost. Therefore, the logical next step is the development and use of a method which requires a calibration but is less delicate than the floating-element technique and can also be used in regions of large pressure gradients. The floating-element technique may then be used to calibrate such a relative instrument.

Two methods of skin-friction measurement used in the past which can be developed further are the Stanton tube technique and the method of measuring local heat transfer. A few measurements using the Stanton tube in high-speed flow and in large pressure gradients have been made by Hakkinen (an unpublished paper). The heat-transfer method used by Fage and Falkner (ref. 3) in laminar boundary layers and by Ludwieg (ref. 4) and by Ludwieg and Tillmann (ref. 5) in turbulent boundary layers will be discussed in this report.

Fage and Falkner as well as Ludwieg have demonstrated that the method is feasible for subsonic laminar and turbulent flow and in regions of large pressure gradient. It is the purpose of the present study to investigate the application of this method to high-speed flow. The basic requirements and range of applicability for such an instrument are discussed. At GALCIT an attempt has been made to develop an instrument capable of being calibrated in a laminar boundary layer and used in a turbulent boundary layer. After considerable time was spent with elaborate devices it was found that a simple hot-wire embedded in the solid surface meets these requirements. Although measurements in high-speed flow have not yet been performed, the theoretical discussion and the results of the low-speed-flow measurements show that this instrument has very attractive features and that further development is well warranted.

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SYMBOLS

C_f	friction coefficient, $\tau_w \Big/ \left(\frac{1}{2}\rho U^2\right)$
c_p	fluid specific heat at constant pressure
f	function in equation (15)

L	streamwise dimension of heated element
Nu	Nusselt number, $q_w \xi / \lambda \Delta T$
Pr	Prandtl number, $c_p \mu / \lambda$
p	pressure
Q_w	total heat flux from element per unit time per unit of cross-stream width
q_w	heat flux per unit area of element per unit time at wall
R_L	Reynolds number based on L, $\rho UL/\mu$
R_ξ	Reynolds number based on ξ, $\rho U \xi / \mu$
T	temperature (also used for plate and element)
t	time
U	free-stream value of u
u	velocity in x-direction
x	downstream coordinate; origin at leading edge of flat plate
y	coordinate normal to plate
ϑ	thickness of thermal layer
θ	thickness of laminar sublayer
δ	thickness of laminar boundary layer
λ	thermal conductivity
μ	coefficient of viscosity
ν	kinematic viscosity
ξ	x-coordinate of upstream edge of heated element

$\pi \equiv \left(\vartheta / 2\tau_w\right)(dp/dx)$

ρ fluid density

τ_w shearing stress at solid surface

ω exponent of viscosity-temperature law

Subscript:

w at wall

PRINCIPLES OF HEATED ELEMENT

Fage and Falkner (ref. 3) and Ludwieg (ref. 4) have given somewhat elaborate theoretical analyses of the heated-element technique of shear measurement, and the recent papers of Lighthill (ref. 6) and Chapman and Rubesin (ref. 7) are also useful. However, it seems preferable here to work out the pertinent features of the method from simple similarity considerations, since this treatment can be extended easily to flow at high speed and to flow with pressure gradients. In working with dimensional and similarity conditions it will of course not be possible to obtain the numerical values of dimensionless coefficients. This fact, however, is of minor importance since problems such as heat leakage will complicate the boundary conditions for an actual element and in most cases will make calibration mandatory.

Fundamentally, the heated-element technique as shown in figures 1 and 2 makes use of the well-known resemblance between the diffusion of heat and of vorticity[1] near a solid surface, the former being responsible for the heat transfer and the latter for the shear. From the surface of a solid body in viscous flow, vorticity diffuses into the flow forming the boundary layer. If the surface temperature of the body is raised above the recovery temperature, heat will diffuse into the flow. The diffusion process consists of continuous conduction of heat or vorticity from the wall into the stream, with simultaneous removal downstream by the external flow.

Flow With Zero Pressure Gradient

Consider first a flat plate extending downstream from $x = 0$ with an insulated surface and therefore a uniform temperature up to a point $x = \xi$. At ξ the surface temperature is raised by an amount ΔT and kept at this level for $x > \xi$.

[1]There is no difficulty in extending the considerations to mass transfer as well.

If μ denotes the viscosity, ρ the density, and U the free-stream velocity, then τ_W, the shearing stress at the wall, can be expressed in terms of a viscous-boundary-layer thickness δ by

$$\tau_W \propto \mu \frac{U}{\delta} \tag{1}$$

with

$$\delta \propto \sqrt{\nu \frac{x}{U}} \tag{2}$$

The heat transfer at the wall q_W, which begins at $x = \xi$, can be written in an analogous way in terms of the temperature difference ΔT and the heat-conduction coefficient λ

$$q_W \propto \lambda \frac{\Delta T}{\vartheta} \tag{3}$$

where ϑ denotes a thermal-boundary-layer thickness which arises from a process identical to the one determining δ, namely, conduction of heat into the fluid and transport downstream. The coefficient equivalent to ν is the ratio $\lambda/\rho c_p$. Hence, by analogy with equation (2),

$$\vartheta \propto \sqrt{\frac{\lambda}{c_p \rho} \frac{x - \xi}{u(\vartheta)}} \tag{4}$$

where the velocity $u(\vartheta)$ has now to be evaluated at the edge of the thermal layer. It should be recalled that equation (2) is often based upon an argument of Lord Rayleigh's using the analogy with the spreading of heat. Rayleigh discusses the problem of an infinitely extended flat plate set impulsively into motion with constant velocity U at time $t = 0$. For an incompressible fluid the equations of motion reduce then to

$$\frac{\partial u}{\partial t} = \nu \nabla^2 u$$

If the plate is suddenly heated to a temperature difference ΔT at time $t = 0$, the corresponding equation for T is:

$$\frac{\partial T}{\partial t} = \frac{\lambda}{c_p \rho} \nabla^2 T$$

Hence the layer of fluid affected by the spreading of vorticity or heat, respectively, is at a time t

$$\delta \propto \sqrt{\nu t}$$

and

$$\vartheta \propto \sqrt{\frac{\lambda}{c_p \rho} t}$$

Equations (2) and (4) follow by considering the time taken by a fluid particle to travel from the origin of the vorticity or heat spreading to the point x as the time t in the nonstationary case. The only new feature in equation (4) is the use of a variable outside velocity $u(\vartheta)$. That is, the existing boundary-layer flow is considered to be a given external flow field into which the heat from the wall diffuses. This concept will prove quite useful for the present considerations and is believed to be applicable to many other problems of a similar type. The region near the wall will be of primary importance for the heat transfer so that within limitations discussed later $u(\vartheta)$ can be replaced by the first term of a series starting from the wall.[2] That is, since the velocity at the wall is zero,

$$u(\vartheta) \approx \left(\frac{\partial u}{\partial y}\right)_w \vartheta + \ . \ . \ . \approx \frac{\tau_w}{\mu} \vartheta \tag{5}$$

Inserting equation (5) into equation (4) yields immediately

$$\vartheta^3 \propto \frac{\lambda \mu}{\rho c_p} \frac{x - \xi}{\tau_w} \tag{6}$$

[2]Fage and Falkner (ref. 3), Ludwieg (ref. 4), Lighthill (ref. 6), and others have also used this approximation.

and the heat flux at the wall becomes, from equation (3),

$$q_w \propto \lambda\Delta T\left(\frac{c_p\rho}{\mu\lambda}\right)^{1/3}\left(\frac{\tau_w}{x - \xi}\right)^{1/3} \tag{7}$$

Introducing a Nusselt number Nu based on the length ξ, equation (7) can be written in terms of the local skin-friction coefficient $C_f = 2\tau_w/\rho U^2$, the Prandtl number $Pr = c_p\mu/\lambda$, and the Reynolds number $R_\xi = \rho U\xi/\mu$:

$$Nu(x,\xi) = \frac{q_w\xi}{\lambda\Delta T} \propto \left(PrR_\xi{}^2C_f\right)^{1/3}\left(\frac{\xi}{x - \xi}\right)^{1/3} \tag{8}$$

From equations (7) and (8) one may also obtain formulas for the integrated heat transfer from a heated strip extending from ξ to $\xi + L$, say; thus

$$Q_w(\xi,L) \propto \left(\frac{c_p\rho}{\mu\lambda}\right)^{1/3}\lambda\Delta T\int_\xi^{\xi+L}\left(\frac{\tau_w}{x - \xi}\right)^{1/3}dx \tag{7a}$$

and

$$Nu(\xi,L) = \frac{Q_w}{\lambda\Delta T} \propto \left(PrR_\xi{}^2\right)^{1/3}\int_\xi^{\xi+L}\frac{C_f{}^{1/3}\,dx}{\xi^{2/3}(x - \xi)^{1/3}} \tag{8a}$$

For the heat transfer from a short strip, that is $L \ll \xi$, $\tau_w(x)$ or $C_f(x)$ will be nearly constant in the range of integration and hence equations (7a) and (8a) become simply

$$Q_w(L,\xi) \propto \left(\frac{c_p\rho}{\mu\lambda}\right)^{1/3}\lambda\Delta T\ \tau_w{}^{1/3}L^{2/3} \tag{7b}$$

and

$$Nu(L,\xi) \propto \left(Pr\ C_f\ R_\xi{}^2\ \frac{L^2}{\xi^2}\right)^{1/3} \tag{8b}$$

Equation (8b) is the basic relation used for the determination of skin-friction coefficients from a measurement of heat transfer at a heated strip of length L. Equations (7) and (8) differ somewhat from the result of Lighthill (ref. 6) and from other more elaborate computations, for example, from Eckert (ref. 8). Lighthill's equation corresponding to equation (7a) reads:

$$Q(\xi,L) = 0.73\left(\frac{c_p\rho}{\mu\lambda}\right)^{1/3}\lambda\Delta T\left(\int_{\xi}^{\xi+L} \tau_w^{1/2}\, dx\right)^{2/3}$$

Hence, except for constant factors the difference between Lighthill's equation and equation (7a) is the form of the shearing-stress integral J; that is, from Lighthill's equation

$$J_1 = \left(\int_{\xi}^{\xi+L} \tau_w^{1/2}\, dx\right)^{2/3}$$

as compared with

$$J_2 = \int_{\xi}^{\xi+L} \left(\frac{\tau_w}{x - \xi}\right)^{1/3} dx$$

from equation (7a). For small values of L, τ_w is nearly constant and the integrals agree except for a constant, since both are proportional to $\tau_w^{1/3}\, L^{2/3}$. The difference is thus quite immaterial for the purpose of the present paper; it arises from the fact that the simple relation of equation (4) for ϑ does not take into account that u, because of the thickening of the boundary layer, depends on x.

On the other hand, it should be kept in mind that the reasoning leading to equation (7) or (8) is based on local considerations only. Thus equations (7) and (8) are valid in both the laminar and the turbulent boundary layers as long as approximation (5) holds. In the turbulent layer this requires the restriction that ϑ be less than the thickness of the laminar sublayer. This condition will be discussed below, after considering the effects of a strong pressure gradient.

Effect of Pressure Gradient

For application of the heat-transfer method near, but not of course at, a separation point (e.g., in shock-wave boundary-layer interaction experiments) it is important to discuss the trend of equations (7) and (8) in a pressure gradient.

The incompressible flow will be considered first. Equations (3) and (4) are still valid locally since the effect of change in velocity profile over the element length is an order smaller than the effect of velocity profile itself, but equation (5) has to be reconsidered. Developing equation (5) to second order yields

$$u(\vartheta) = \frac{\tau_w}{\mu}\vartheta + \left(\frac{\partial^2 u}{\partial y^2}\right)_w \frac{\vartheta^2}{2} + \ldots \tag{9}$$

In a boundary layer the pressure gradient dp/dx is related simply to $\left(\partial^2 u/\partial y^2\right)_w$ by

$$\frac{dp}{dx} = \left(\frac{\partial \tau}{\partial y}\right)_w = \mu\left(\frac{\partial^2 u}{\partial y^2}\right)_w$$

and hence equation (9) can be written

$$u(\vartheta) = \frac{\tau_w \vartheta}{\mu}\left(1 + \frac{\vartheta}{2\tau_w}\frac{dp}{dx} + \ldots\right) \tag{9a}$$

Thus the parameter

$$\pi \equiv \frac{\vartheta}{2\tau_w}\frac{dp}{dx}$$

characterizes the influence of the pressure gradient. If $dp/dx \leqq 0$ the shear τ_w is large and π will be small. Consequently, the relations of the section entitled "Flow With Zero Pressure Gradient" will remain unaltered.

Approaching separation, however, $(\vartheta/\tau_w)(dp/dx)$ will increase indefinitely because τ_w tends to zero. Thus it is necessary to establish the heat-transfer relations for large values of π to determine the effect of pressure gradient. For this case $u(\vartheta)$ becomes independent of τ_W,

$$u(\vartheta) \approx \frac{\vartheta^2}{2\mu}\frac{dp}{dx} \tag{10}$$

and hence from equation (4)

$$\vartheta^4 \propto \left(\frac{\lambda\mu}{c_p\rho}\right)\frac{x - \xi}{dp/dx} \tag{11}$$

Consequently equations (7) and (8) become

$$q_W \propto \lambda\Delta T\left(\frac{c_p\rho}{\mu\lambda}\right)^{1/4}\left(\frac{dp}{dx}\right)^{1/4}(x - \xi)^{-1/4} \tag{12}$$

and

$$Nu(x,\xi) \propto Pr^{1/4}\left(\frac{\rho\xi^3}{\mu^2}\frac{dp}{dx}\right)^{1/4}\left(\frac{\xi}{x - \xi}\right)^{1/4} \tag{13}$$

or

$$Nu(x,\xi) \propto \left(PrR_\xi{}^2\right)^{1/4}\left(\frac{\xi}{\rho U^2}\frac{dp}{dx}\right)^{1/4}\left(\frac{\xi}{x - \xi}\right)^{1/4}$$

The equivalent of equation (8b) would thus read

$$Nu(\xi,L) \propto \left(PrR_\xi{}^2\frac{L^3}{\xi^3}\right)^{1/4}\left(\frac{\xi}{\rho U^2}\frac{dp}{dx}\right)^{1/4} \tag{14}$$

where it is, of course, still assumed that $dp/dy \ll dp/dx$ at the point of measurement.

Application to Turbulent Boundary Layers

The simplest and most direct application to turbulent boundary layers of the heated-element technique, in view of the foregoing simple considerations, follows when the thermal-boundary-layer thickness ϑ is required to be smaller than the laminar sublayer thickness θ of the turbulent boundary layer. In this case the conditions in the incompressible turbulent and laminar boundary layers differ only in the value of τ_w, and hence a single calibration should be sufficient to establish the numerical constant in equation (8b) for application both in the turbulent and laminar case. That this is experimentally feasible is shown presently. However, the pertinent similarity relations are developed first.

Near the wall the velocity distribution in a turbulent boundary layer follows the "universal wall law;" that is, $u(y)$ obeys a relation of the form

$$u(y) = \sqrt{\frac{\tau_w}{\rho}}\, f\left(\frac{y}{\nu}\sqrt{\frac{\tau_w}{\rho}}\right) \tag{15}$$

The extent of the so-called laminar sublayer depends upon the form of the function f. Ordinarily one defines the laminar sublayer thickness θ as the region in which u is a linear function of y; that is,

$$f'' \ll f'$$

although physically it seems better to define θ entirely in terms of energy dissipation, as, for example, one defines a microscale of turbulence. Since only an order-of-magnitude estimate is needed, numerical constants of order unity can be omitted; thus

$$\theta \approx \mu \frac{U}{\tau_w} \tag{16}$$

so that the condition for application of the heated-element technique to the turbulent boundary layer reads

$$\vartheta < \mu \frac{U}{\tau_w} \tag{17}$$

The maximum value of ϑ is reached at the downstream end of the element, that is, at $(x - \xi) = L$, and thus from equation (6)

$$\left(\frac{\lambda\mu}{c_p\rho} \frac{L}{\tau_w}\right)^{1/3} < \mu \frac{U}{\tau_w}$$

or

$$\frac{\rho LU}{\mu} \equiv R_L < \Pr \, C_f^{-2} \tag{18}$$

Hence the Reynolds number based on the length of the heated element has to be less than the Prandtl number divided by the square of the skin-friction coefficient. Using a simple empirical relation for C_f for turbulent flow, equation (18) can be written conveniently in terms of L/ξ. For a turbulent flat-plate boundary layer C_f is approximately given by

$$C_f \propto R_\xi^{-1/5}$$

and equation (18) becomes

$$\frac{L}{\xi} < \Pr R_\xi^{-3/5} \tag{19}$$

It is possible to satisfy this condition experimentally by making the effective length of the element small enough. However, two interesting cases arise. First, there exists also a lower limit for L, beyond which the boundary-layer approximation ceases to be sufficient, since $\partial T/\partial x$ cannot be considered small compared with $\partial T/\partial y$; expressed differently, ϑ is no longer small compared with L. A short discussion of this limitation is given later on. Second, the length of the heated element may be larger than that required by equation (19), and then the flow outside the sublayer will become important. This will affect both the mean-speed distribution and the effective heat conductivity. In this case, while measurement of skin friction by use of the heated element becomes more difficult, it is possible that the method may become useful for the determination of turbulent exchange coefficients near the wall.

Effect of Compressibility

Since the purpose of the present discussion is primarily to formulate a theory for a skin-friction measuring instrument, it is possible to restrict the temperature difference ΔT to values small enough to neglect the variations of μ, ρ, λ, and so forth due to ΔT. However, in order to extend the previous equations to high-speed flow, variations of μ, ρ, λ, and so forth due to variations in mean temperature in the boundary layer must be considered.

The quantity ΔT will evidently always denote the temperature difference from recovery temperature, except when heat transfer is present initially; the analysis can of course be extended to this case if desired. To estimate the Mach number effect it is thus necessary to estimate the temperature difference across a layer of thickness ϑ in the original flow. Since ϑ is always to be kept small compared with the boundary-layer thickness δ this difference can usually be expected to be small; hence simple and rather crude estimates will suffice.

Considering flow of a perfect gas, the energy integral may be written in the form valid for $Pr = 1$ and zero heat transfer,

$$\frac{1}{2} u^2 + c_p T = \text{Constant} \tag{20}$$

Writing

$$T(\vartheta) = T_w + \left(\frac{\partial T}{\partial y}\right)_w \vartheta + \left(\frac{\partial^2 T}{\partial y^2}\right)_w \frac{\vartheta^2}{2} + \cdot \cdot \cdot$$

and evaluating the derivatives from equation (20),

$$\frac{T(\vartheta) - T_w}{T_w} \approx - \frac{\vartheta^2}{2c_p T_w} \left(\frac{\partial u}{\partial y}\right)_w^2 \tag{21}$$

or

$$\frac{T_w - T(\vartheta)}{T_w} \approx \left(\frac{R_L C_f{}^2}{4Pr}\right)^{2/3} \frac{U^2}{2c_p T_w}$$

or better

$$\frac{T_w - T(\vartheta)}{T_w} \approx \left(\frac{R_L C_f^2}{4Pr}\right)^{2/3} \frac{\frac{\gamma - 1}{2} M_\infty^2}{1 + \frac{\gamma - 1}{2} M_\infty^2} \tag{22}$$

To be consistent the Prandtl number in equation (22) should be set equal to unity or else M_∞^2 should be multiplied by the appropriate function of Pr accounting for the deviation of the actual energy integral from equation (20).

The effect on the measurements of the temperature difference as expressed in equations (21) or (22) is found from equation (7b), which can be written

$$\frac{Q_w(L,\xi)}{\Delta T \tau_w^{1/3} L^{2/3}} \propto \left(\frac{c_p \rho \lambda^2}{\mu}\right)^{1/3} = c_p Pr^{-2/3} (\rho\mu)^{1/3} \tag{23}$$

The variations in temperature will be in general too small to affect c_p and Pr at all and hence the compressibility effect for the instrument consists essentially of the variation of $\rho\mu$ across the thermal layer. The effect can be easily estimated now either by using approximations to Sutherland's viscosity law or even more simply by using a power law for the $\mu(T)$ relation. Putting

$$\mu = \mu_w \left(\frac{T}{T_w}\right)^\omega \tag{24}$$

the mean value $\overline{(\rho\mu)^{1/3}}$ in the thermal layer is related to the wall value by

$$\frac{\overline{(\rho\mu)^{1/3}}}{(\rho\mu)_w^{1/3}} = 1 + \frac{1 - \omega}{9} \left(\frac{R_L C_f^2}{4Pr}\right)^{2/3} \frac{(\gamma - 1)M_\infty^2}{2 + (\gamma - 1)M_\infty^2} \tag{25}$$

Now for air $\omega \approx 0.76$, hence $(1 - \omega)/9 \approx 0.03$, and therefore the compressibility effect is in most cases negligible since the factor multiplying $(1 - \omega)/9$ will be smaller than unity for a reasonably designed instrument.

Thus, extensions of this method of skin-friction measurement to compressible fluid flow, while not trivial, should be perfectly feasible. It should be noted that the discussion of pressure-gradient effects remains valid for first approximations. For compressible fluid flow the relation between pressure gradient dp/dx and $(\partial^2u/\partial y^2)_w$ reads

$$\left(\frac{\partial^2 u}{\partial y^2}\right)_w = \frac{1}{\mu_w}\frac{dp}{dx} - \frac{\tau_w}{\mu_w}\left(\frac{\partial \log_e \mu}{\partial y}\right)_w \tag{26}$$

with

$$\mu \propto T^{\omega}$$

This yields

$$\left(\frac{\partial^2 u}{\partial y^2}\right)_w = \frac{1}{\mu_w}\frac{dp}{dx} - \frac{\omega\tau_w}{T\mu_w}\left(\frac{\partial T}{\partial y}\right)_w \tag{27}$$

For the undisturbed flow $(\partial T/\partial y)_w = 0$ and hence the relation between $\left(\partial^2 u/\partial y^2\right)_w$ and dp/dx is nearly the same as for incompressible flow.

Limitations of Boundary-Layer-Type Analysis

The relations developed so far for the characteristics of the heated element are boundary-layer-type relations. This is implied in equation (4), which gives ϑ in terms of $x - \xi$ and u, and also in the tacitly assumed fact that downstream conditions are immaterial so that an element of length L will behave like the initial L units of a semi-infinite element. The restrictions so far imposed upon L are such as to require that L be small. However, it is evident that a limit will be reached beyond which L cannot be decreased without a violation of the boundary-layer-type flow. To estimate this limit the relation between L and ϑ can be written down. The boundary-layer-type study then requires $L \gg \vartheta$, that is,

$$\left(C_f Pr R_L{}^2\right)^{1/3} \gg 1 \tag{28}$$

or, comparing equation (28) with equation (8b), simply

$$Nu(L,\xi) \gg 1 \tag{28a}$$

This gives a lower limit for Nu. The most stringent upper limit is given by equation (18) for the case of the turbulent boundary layer. Using equations (18) and (28a) the range for which the instrument is applicable in the form described here is given by

$$\frac{Pr}{C_f} > Nu(L,\xi) \gg 1 \tag{29}$$

EXPERIMENTAL PROCEDURES AND RESULTS

Heated Element

Considerable time was spent in developing various types of elements and methods of measuring heat transfer. The final experimental arrangement is described below, while a few remarks concerning other methods are included in the appendix.

An ebonite plate, with a 0.0005-inch-diameter platinum wire cemented in a groove in the surface, was held in a free air jet. Turbulent layers were established by using a trip wire near the leading edge, or in the case of the lowest turbulent shearing stress by supporting the trip wire just off the surface of the plate and allowing it to vibrate in order to trip the flow. Laminar layers were obtained with the plate clean. Velocity profiles were obtained with a similar platinum wire, mounted as an ordinary hot-wire, carried on a micrometer traverse. The apparatus is sketched in figure 3. Shear stress was calculated from the slope of the velocity profile at the wall (in the laminar sublayer in the case of turbulent boundary layers).

A first attempt to use the hot-wire itself as the surface-heated element by touching it to the wall proved unsuccessful, possibly because of slight irregularities in the ebonite surface. The conduction of heat to the wall is always a source of trouble when a hot-wire is used very close to a surface, and it was thought that by continuing the readings right up to the wall, the wire should behave like one embedded in the wall. An example of the profile so obtained is given in figure 4.

With the flush-mounted wire, readings of heat flow per unit rise in temperature of the wire were obtained and plotted against the one-third power of the shearing stress computed from the hot-wire velocity profiles. Figure 5 shows that it is possible to obtain reasonable consistency. The scatter of points in figure 5 is largely connected with the measurement of shearing stress by the slope of the profile near the wall. The scatter can probably be reduced considerably by a calibration with a floating element.

Instrumentation

For velocity-profile work the hot-wire and a half-ohm standard resistance were connected in one arm of a Shallcross Wheatstone Bridge and resistance was maintained constant to four-figure accuracy. A Leeds and Northrup type K-2 potentiometer connected across the half-ohm standard resistance gave the current reading to four significant figures. Wire temperature was computed from wire resistance, while air temperature was measured on a mercury thermometer in the air stream to an accuracy of at least $1/10^{\circ}$ C.

For measurements with the surface wire the same electrical apparatus was used, this wire being substituted for the hot-wire probe. During these measurements considerable time was allowed for the readings to stabilize.

Hot-Wire Calibration by Shedding Frequency of a Cylinder

A point of interest may be a hot-wire calibration which was accomplished using the vortex-shedding frequency of a cylinder to measure wind velocity, as described by Roshko in reference 10. Figure 6 shows the high accuracy attainable by this method, even to the checking of the free-convection point (zero velocity). The lowest speed read by means of the shedding frequency was 48 centimeters per second, which with alcohol of 0.81 specific gravity would correspond to a pitot-static reading of less than 0.002 centimeter of alcohol. At such speeds the shedding-frequency method is still highly accurate, with proper choice of cylinder diameter, so that accurate calibration was possible down to speeds as low as almost all those used in determining velocity gradients at the surface of the plate.

All the work was done with relatively cool wires; the wire temperature never exceeded 100° C, and air temperature was in the region of 20° C. Throughout the work slow variations of temperature were encountered so that the results represent what could reasonably be expected under ordinary operating conditions in most wind tunnels.

For practical applications a much neater installation would be desirable, possibly a plug of insulating material, such as ebonite, inserted in the model surface, the wire being cemented in a groove and properly smoothed off at the surface. Here one must bear in mind that the extent of the insulating material has to be sufficient to make any conduction to the model negligible, so that a single calibration will hold good.

At low speeds the whole of the model should reach stagnation temperature, so that heat loss to the model should be sensibly independent of the type of boundary layer at the point of study, where the only appreciable deviation from stagnation temperature occurs.

When conditions of varying surface temperature arise in the model, it is then, of course, important that the conditions of heat loss be the same for both calibration and actual use. In general, this means that the "instrument" must be subjected to conditions near the point of measurement only and must be separated from the remainder of the model. Air being a better insulator than any solid by a factor of about 60, it is clearly advisable to mount the instrument plug in the surface, leaving an adequate air space surrounding the plug as much as possible. The wire then settles down to the local recovery temperature and when heated is raised an amount ΔT above this temperature.

Evaluation of Effective Length of Heated Element

The heat supplied by an electrically heated element diffuses into the solid surface in which the element is embedded. Hence, the length L, which enters the simple relations assuming a uniformly heated element, has to be replaced by a suitable mean value. For the measurements themselves this refinement is not necessary since the instrument is calibrated. However, to check whether inequality (28) is satisfied, one has to estimate L. This may be done from the slope of the straight line giving Q_W, or a related quantity, as a function of $\tau_W^{1/3}$ (fig. 5). For the case of the hot-wire embedded in the ebonite surface, L was found to be 0.5 centimeter. Inequality (29) is satisfied throughout the present experiments, the ratio Pr/C_f being of the order of 1,000 and $Nu(L,\xi)$ being of the order of 30.

CONCLUDING REMARKS

The experiments reported here show that it is possible to use a simple hot-wire embedded in an insulating surface to obtain local skin-friction coefficients in low-speed flow. A single calibration giving the heat loss of the wire in terms of the shearing stress on the surface is found to hold for laminar and turbulent boundary layers. The theoretical discussion defines the requirements for such an instrument and indicates the possibility of extending the technique to high-speed flow and large pressure gradients. The basic relations for the total heat loss Q_W from the heated element, when kept at a temperature difference ΔT from the equilibrium temperature of the solid surface, may be expressed in terms of the Prandtl number Pr, the skin-friction coefficient C_f, and a Reynolds number R_L based on the effective length L of the element:

$$\frac{Q_w}{\lambda \Delta T} \propto Pr^{1/3} C_f^{1/3} R_L^{2/3} \qquad \text{for } \frac{dp}{dx} \leqq 0$$

$$\frac{Q_w}{\lambda \Delta T} \rightarrow Pr^{1/4} \left(\frac{L}{\rho U^2} \frac{dp}{dx} \right)^{1/4} R_L^{1/2} \qquad \text{near laminar separation}$$

Here x is the thermal conductivity, dp/dx is the pressure gradient, U is the free-stream velocity, and ρ is the fluid density. In order to allow a single calibration for laminar and turbulent boundary layers, while at the same time satisfying the boundary-layer approximations which lead to the expressions above, the inequality

$$1 \ll \frac{Q_w}{\lambda \Delta T} < \frac{Pr}{C_f}$$

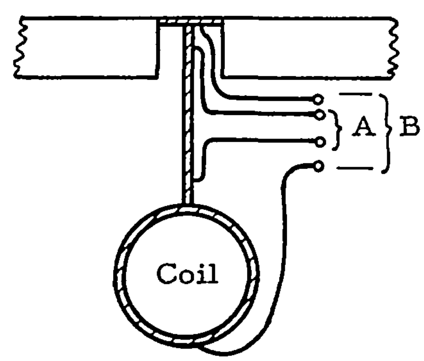
must be satisfied.

California Institute of Technology,
Pasadena, Calif., July 15, 1953.

APPENDIX

DIFFERENT FORMS OF HEATED ELEMENT

Prior to the use of a wire mounted in the surface, larger heated elements were investigated; the latter, although sensitive to the type of boundary layer to which they were exposed, were perfectly capable of calibration even when solidly mounted in an ebonite plate without any effort to avoid losses to the plate. Two methods of heating were used. The first, following Ludwieg's example, used a heating helix wound on a glass rod and inserted in a brass block, which was then lapped flush with the surface of the plate. The second method employed a thin plate, lapped flush with the surface and fed with heat by an iron strip terminating in a loop of iron some distance below the plate, as shown in the sketch. This loop was heated by induction, a coil being placed inside it and fed with 1/2-megacycle-per-second alternating current from an oscillator. The rate of heat flow to the element was measured by thermojunctions employing constantan wire brought out to terminals A, while the surface-temperature rise above the equilibrium plate surface temperature was obtained by means of a thermojunction formed by bringing a constantan wire from the surface element and an iron wire from the bottom of the ring to terminals B. A reference junction was placed in the surface of the plate at some distance from the heated element. The object of induction heating was to permit a glass housing to be built around the system and evacuated in order to cut down extraneous losses.



Although no attempt was made to elaborate upon the system, it had a reasonably fast response, and transition from laminar to turbulent boundary layer could be detected immediately.

The possibility of mounting a very thin but well-insulated element flush with the surface was considered for the purpose of making dynamic measurements. By induction heating the element could be raised to a steady temperature, as measured by fine thermocouple wires in the element, and the field could then be switched off. The initial rate of cooling would give the rate of heat transfer at the initial temperature, and, by repeated cycling, a steady pattern could be obtained on an oscilloscope.

If m is the mass of the element, σ is the specific heat of the element material, T_W is the temperature of the element, H is the heat content of the element, and Q_W is the rate of heat transfer to the boundary layer, then

$$Q_W = - \frac{dH}{dt} = -m\sigma \frac{dT_W}{dt}$$

when no energy is being supplied.

A study of the element dimensions required, even for very low-frequency cycling of the process, shows that for most ordinary values of skin friction the element would have to be so thin that construction would be quite impossible. In fact, to obtain sufficient mechanical strength in the element to permit lapping the junction with the plate, so much mass is required that the transient response would be very slow indeed; on the other hand, a single cooling cycle would be of little use for accurate measurement of heat-transfer rates.

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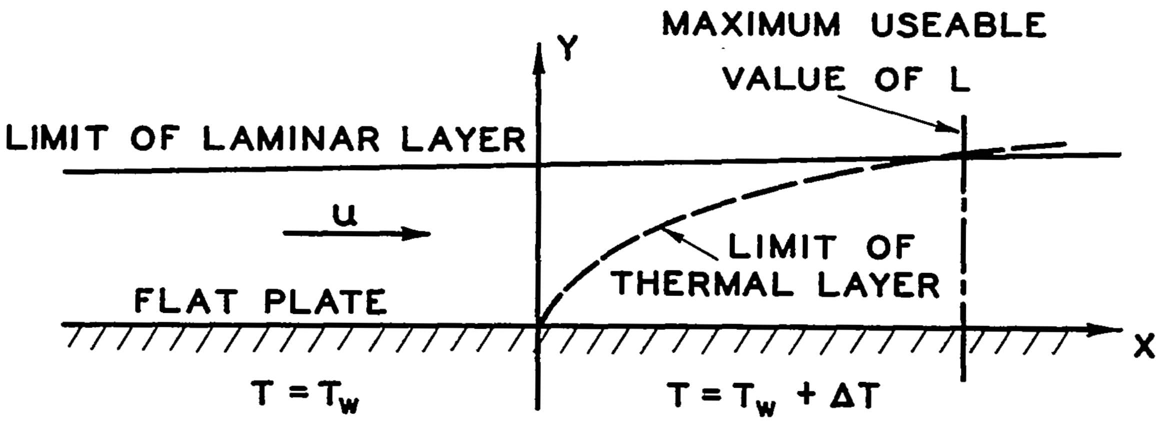

Figure 1.- Heating of fluid by an element.

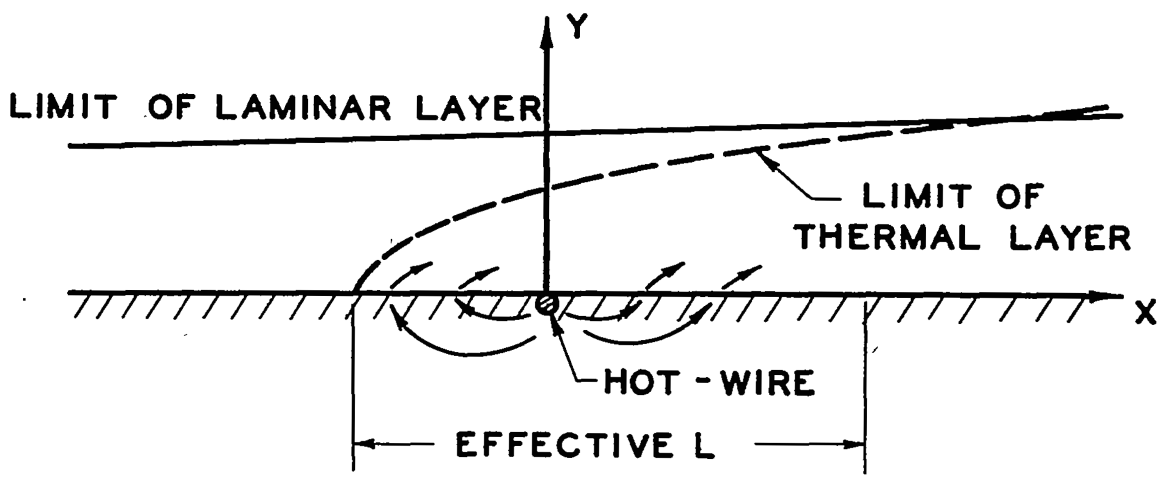

Figure 2.- Heating of fluid by a wire in surface.

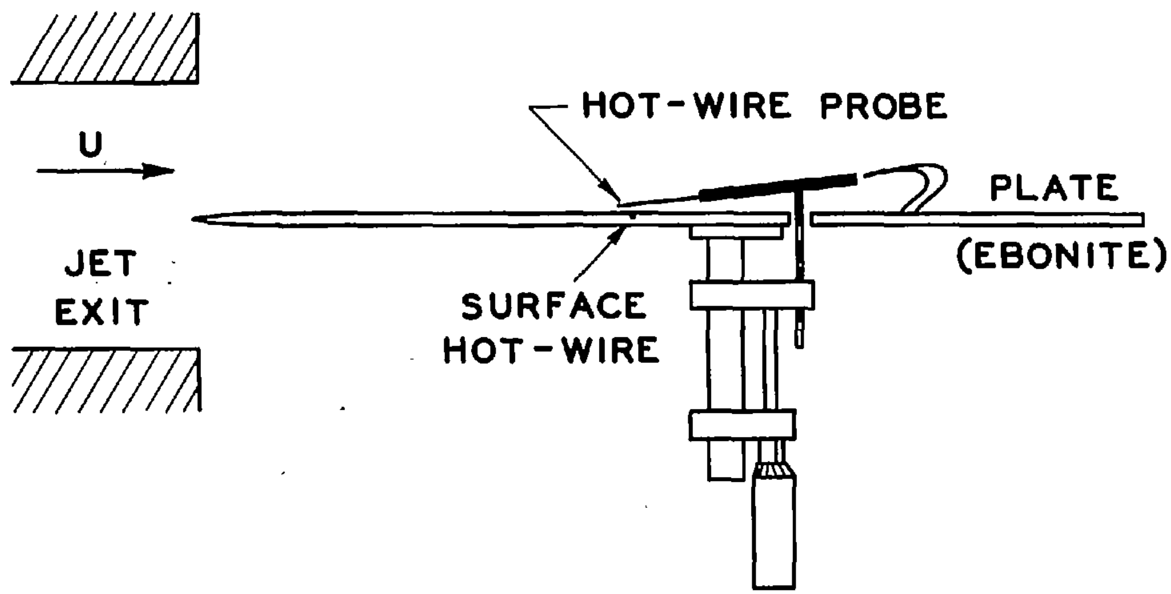

Figure 3.- Sketch of apparatus.

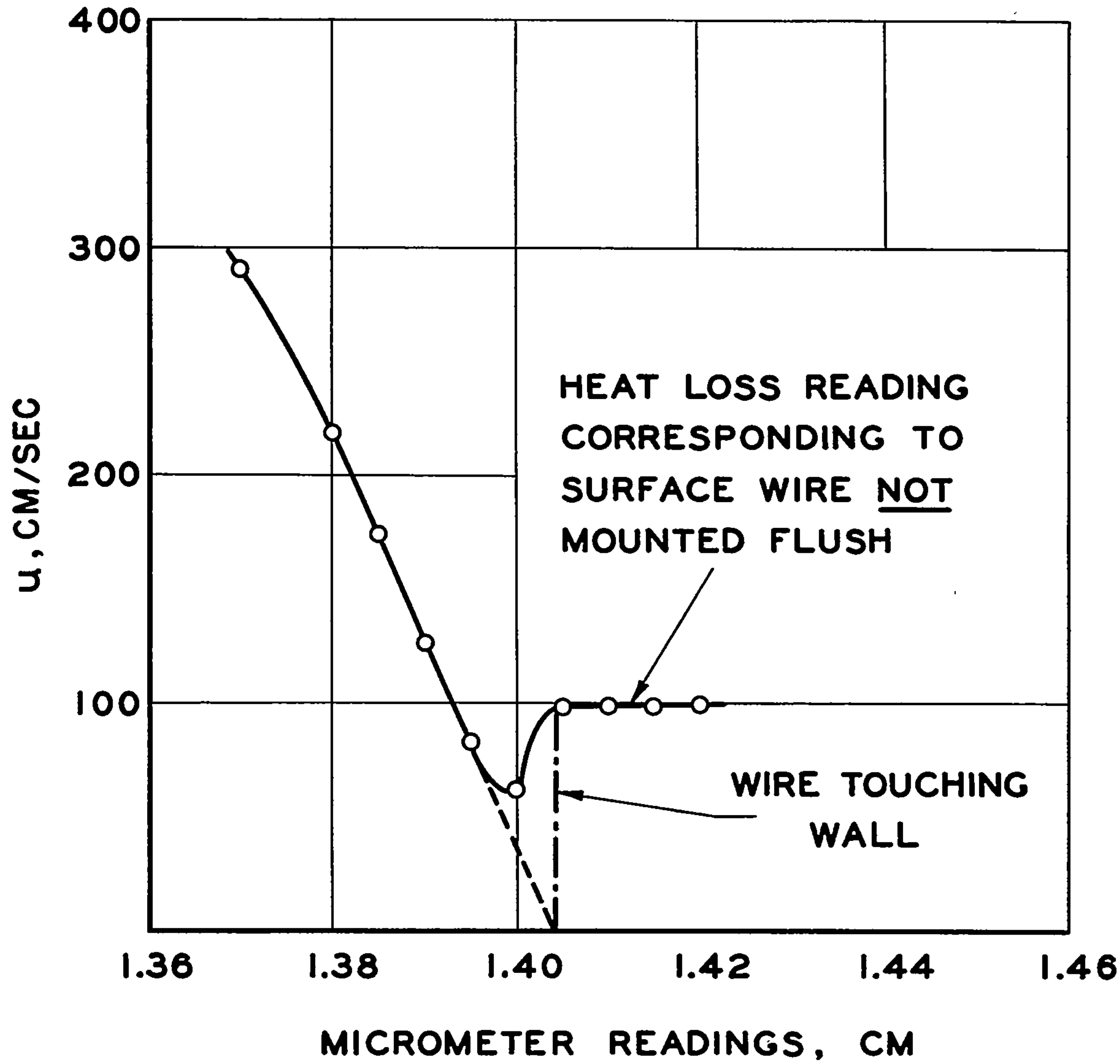

Figure 4.- Extended velocity profile illustrating effect of touching wire to wall.

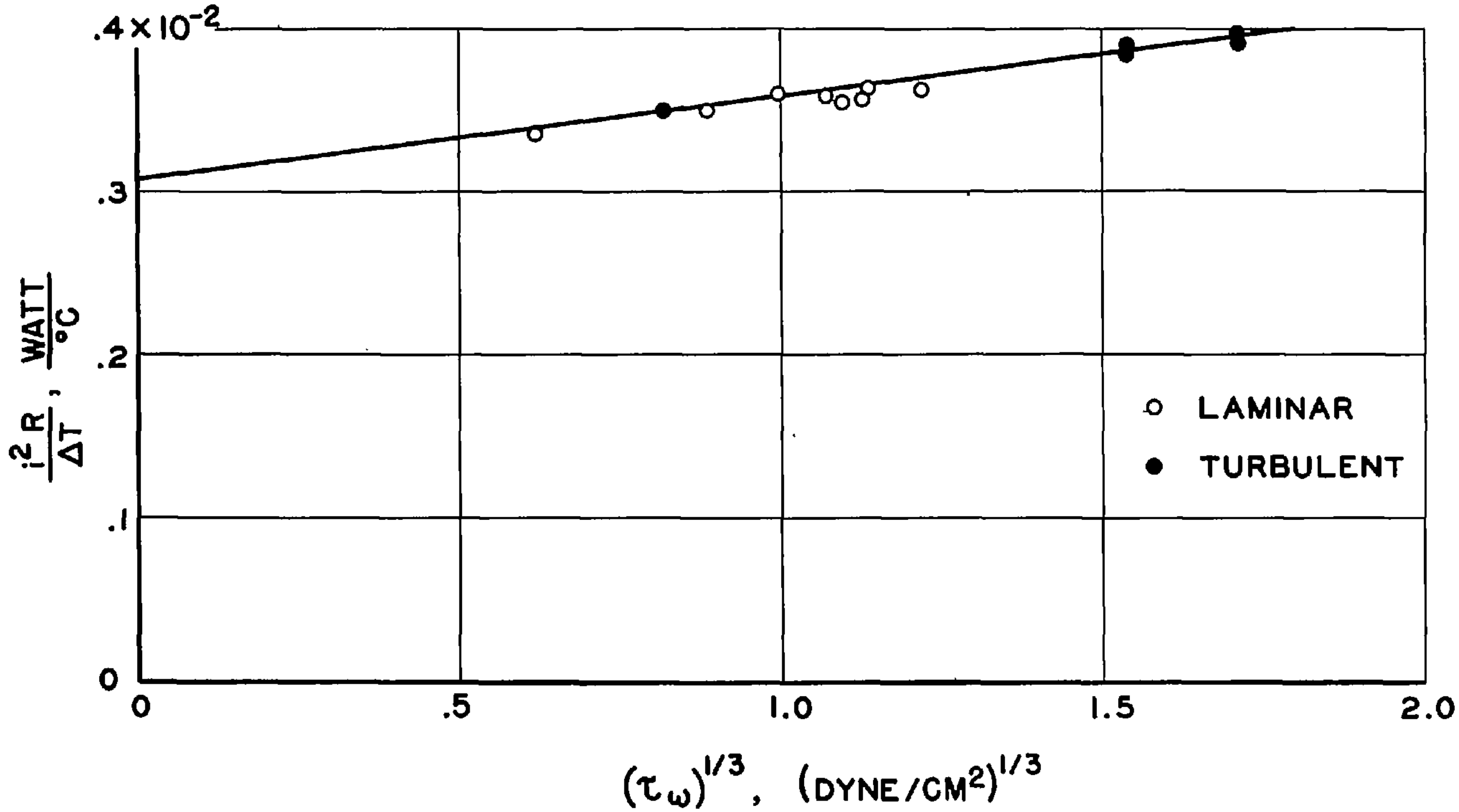

Figure 5.- Heat loss from a wire cemented flush with surface.
i, hot-wire current; R, resistance.

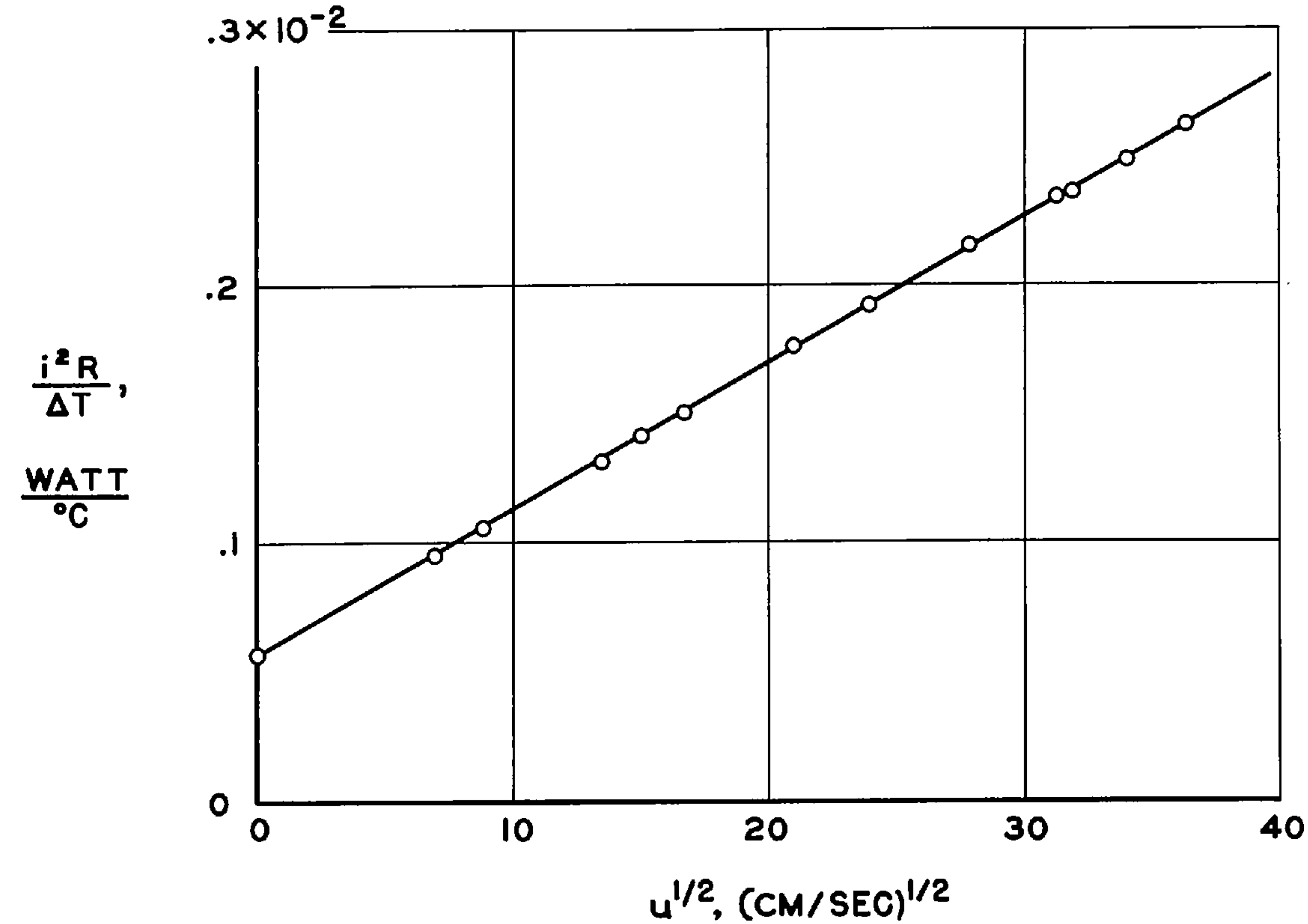

Figure 6.- Hot-wire velocity probe calibration using shedding frequency of cylinder as standard. i, hot-wire current; R, resistance.

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